Unrestricted Bridge (U-Bridge) Designs for Settings with Non-Deterministic Data

Bradley Jones

June 2026

Joint work with David Steinberg
of Tel Aviv University

Quality and Reliability Engineering International WILEY

SPECIAL ISSUE ARTICLE OPEN ACCESS

Unrestricted Bridge (U-Bridge) Designs for Settings with Non-deterministic Data

Bradley Jones[1] | David M. Steinberg[2]

To cite:

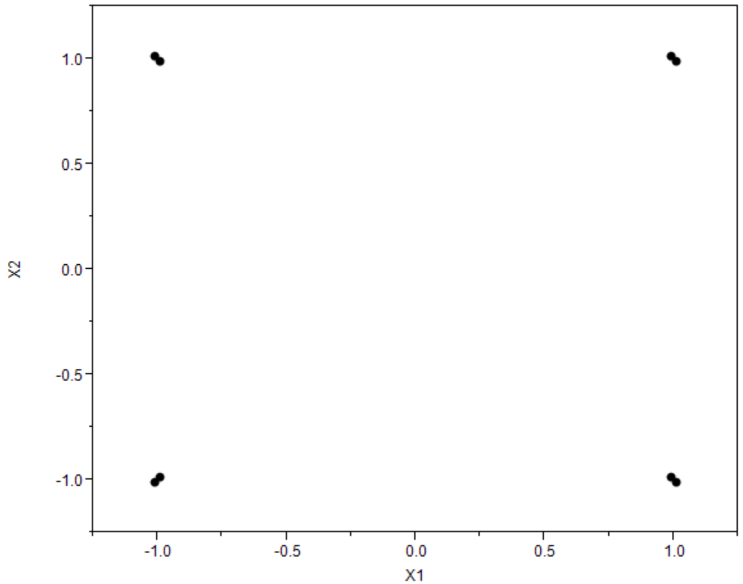
B. Jones, D. M. Steinberg, Unrestricted bridge (U-Bridge) designs for settings with non-deterministic data, Quality and Reliability Engineering International 41 (2025) 3405–3414. doi:https://doi.org/ 10.1002/qre.70050.

Motivation – Balance Model Robustness & Precise Parameter Estimation

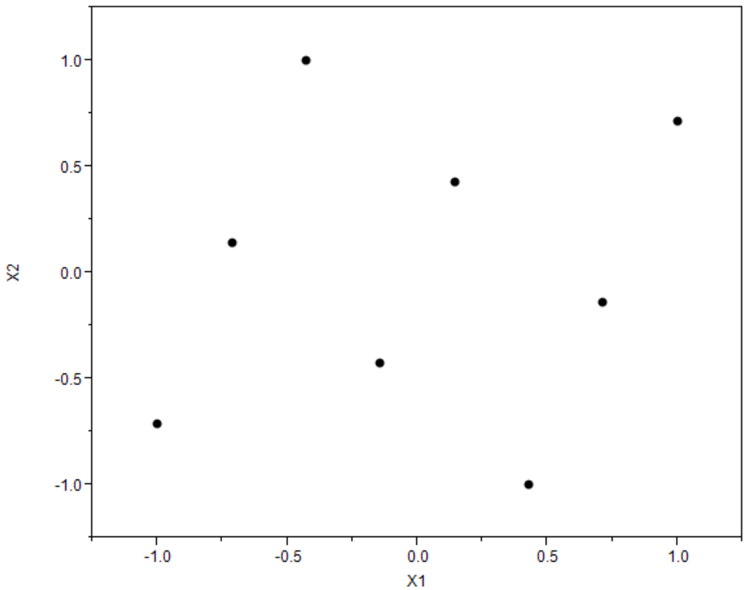
Latin Hypercube designs (space filling) are the default for deterministic computer simulation studies.

Factorial designs are standard for modeling systems with substantial noise.

Factorial vs. Latin Hypercube



Factorial



Latin Hypercube

Jitter added to reveal replication

Pros and Cons

Factorial (Two Level)

Pros

1. Good for precise parameter estimation
2. Good prediction for 1^{st} order polynomials

Cons

1. Cannot detect curvature
2. Replication is useless if there is no noise

Latin Hypercube

Pros

1. Good for detecting and modeling curvature
2. Good for integral approximation

Cons

1. Inefficient estimation if there is noise
2. Less powerful for modeling interactions

Latin Hypercube Variance Efficiency

Number of Factors	Number of Runs	Relative D-Efficiency of LHC (%)
2	16	60.3
2	32	57.6
3	16	54.3
3	32	52.2
4	16	51.1
4	32	48.7
5	16	45.7
5	32	45.2

Average relative efficiency for a main effects model comparing LHC and 2-level factorial and fractional factorial designs.

Coefficient Variance Comparison LHC vs FF

Effect	Variance	Variance
Intercept	0.036	0.031
X1	0.106	0.031
X2	0.103	0.031
X3	0.108	0.031
X4	0.112	0.031
X5	0.109	0.031
X6	0.104	0.031
X1*X2	0.384	0.031
X1*X3	0.397	0.031
X1*X4	0.398	0.031
X1*X5	0.341	0.031
X1*X6	0.482	0.031
X2*X3	0.353	0.031
X2*X4	0.302	0.031
X2*X5	0.411	0.031
X2*X6	0.422	0.031
X3*X4	0.441	0.031
X3*X5	0.464	0.031
X3*X6	0.368	0.031
X4*X5	0.407	0.031
X4*X6	0.38	0.031
X5*X6	0.291	0.031

6 Factors 32 Runs
Interactions model

Bias vs. Variance Considerations

When the response function is deterministic you are only concerned about the model bias induced by using a simpler surrogate model.

Variance of parameter estimates is a primary concern when the noise is larger than the signal.

When the noise is smaller than the signal, then it makes sense to consider both variance and bias in design construction.

Bridge Design (2015)

Goals

1. High D-efficiency for low order polynomial models
2. No replication in projection for GP model fitting

Implementation

Algorithmic – find a design that maximizes efficiency subject to a minimum distance in every direction for each point pair.

Reference:

Bradley Jones, Rachel T. Silvestrini, Douglas C. Montgomery, and David M. Steinberg. Bridge designs for modeling systems with low noise. Technometrics, 57(2):155–163, 2015.

Issues with Bridge Designs

1. Original algorithmic implementation was slow and could fail to find the best design.
2. Thanks to E. Benková, R. Harman, and W. G. Müller, their implementation was faster and more efficient for fitting polynomial models.
3. However, the restriction requiring a minimum distance is arbitrary

E. Benková, R. Harman, and W. G. Müller, "Privacy Sets for Constrained Space-Filling," Journal of Statistical Planning and Inference 171 (2016): 1–9.

What If?

Since the restriction to a minimum distance between points in every dimension is arbitrary, why not remove it?

Introducing Unrestricted Bridge Designs

We call them U-Bridge Designs.

U-Bridge Designs are space filling designs in that replication is not allowed.

U-Bridge Designs do not require that points be a minimum distance from each other in each dimension.

Two factor sample U-Bridge design

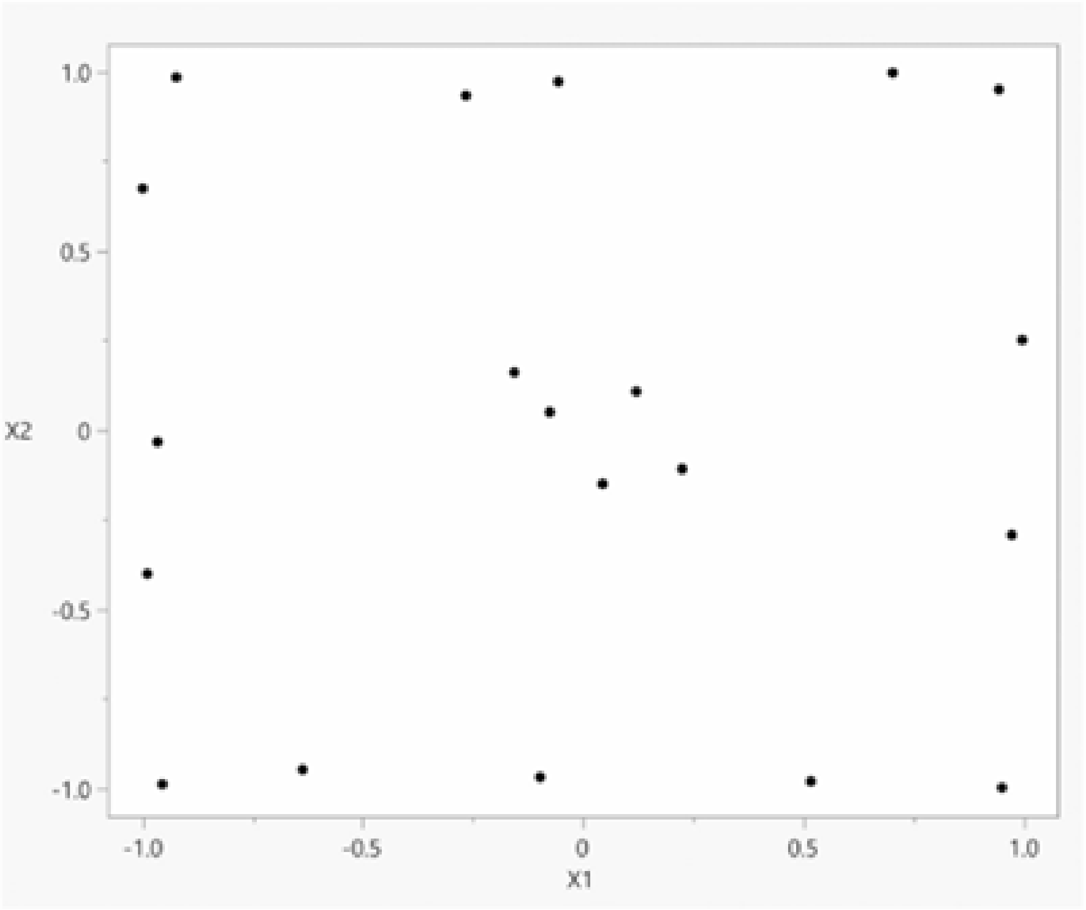


FIGURE 1 | U-bridge design with $k = 2$ and $n = 20$.

Design Construction – part 1

The U-Bridge design algorithm starts with generating a Fast Flexible Filling Design (FFF design)

Building a FFF design has three steps.

1. Generate a random sample of a very large number of points in the design space.

2. Cluster the points into as many clusters as the number of runs in the FFF design to be created.

3. Choose a point from each cluster to maximize some measure of space filling quality. Currently the JMP implementation uses the MaxPro criterion, which guarantees no replication.

Design Construction – part 2

The U-Bridge design algorithm has two steps

1. Create a FFF design with many times more points than are desired for the U-Bridge design.
   a) We recommend the FFF to have 20x the number of points than the U-Bridge design
   b) For a 20 run U-Bridge Design make a 400 run FFF design
2. Use the FFF design as a candidate set for generating the U-Bridge design using an optimal design criterion.
   a) We recommend designing for the full quadratic model
   b) We use the I-optimality criterion to minimize the average prediction error

R.Lekivetz and B.Jones, "Fast Flexible Space-Filling Designs for Non-rectangular Regions," Quality and Reliability Engineering International 31 (2015): 829–837.

Bridge Design – Published Article online

Quality and Reliability Engineering International

WILEY

SPECIAL ISSUE ARTICLE OPEN ACCESS

Unrestricted Bridge (U-Bridge) Designs for Settings with Non-deterministic Data

Bradley Jones[1] | David M. Steinberg[2]

[1]Independent Consultant, Cary, NC, USA | [2]Department of Statistics and Operations Research, Tel-Aviv University, Tel Aviv, Israel

Correspondence: David M. Steinberg (dms@tauex.tau.ac.il)

Received: 2 July 2025 | **Revised:** 11 August 2025 | **Accepted:** 19 August 2025

Funding: The authors received no specific funding for this work.

Keywords: bridge design | gaussian process regression | optimal design | polynomial models | space-filling design

Download for free using the link below:

https://doi.org/10.1002/qre.70050

Comparison – LHD vs U-Bridge

TABLE 1 | Relative efficiency of maximin Latin hypercube designs to U-bridge designs.

Factors	Runs	Criterion *D*	Criterion *A*	Criterion *I*
2	10	0.583	0.457	0.621
2	20	0.645	0.583	0.715
3	15	0.532	0.420	0.571
3	30	0.627	0.585	0.676
4	20	0.533	0.459	0.578
4	30	0.508	0.438	0.539

For fitting full quadratic polynomial models

Comparison – MaxPro vs U-Bridge

TABLE 2 | Relative efficiency of MaxPro designs to U-bridge designs.

		Criterion		
Factors	**Runs**	***D***	***A***	***I***
2	10	0.632	0.509	0.675
2	20	0.691	0.625	0.756
3	15	0.638	0.548	0.688
3	30	0.724	0.683	0.774
4	20	0.672	0.574	0.692
4	30	0.439	0.351	0.426

For fitting full quadratic polynomial models

Comparison – Bridge vs U-Bridge

TABLE 3 | Relative efficiency of bridge designs (with a constraint of $1/(n-1)$) to U-bridge designs.

		Criterion		
Factors	**Runs**	***D***	***A***	***I***
2	10	0.934	0.840	0.894
2	20	0.807	0.738	0.851
3	15	0.900	0.811	0.872
3	30	0.550	0.514	0.609
4	20	0.847	0.749	0.833
4	30	0.459	0.364	0.429

For fitting full quadratic polynomial models

Nonlinear Test Case Comparisons

We compared 3 test cases from

https://www.sfu.ca/~ssurjano/index.html

1. Wing weight
2. OTL circuit
3. Borehole

We did two types of comparison:

1. Polynomial fit with added noise U-Bridge designs outperformed LHD and MaxPro wrt RMSE

2. GP fit with added noise U-Bridge designs performed slightly worse wrt RMSE

When to use U-Bridge Designs

1. The response data have noise, i.e. replicating a point does not give the same response.
2. Cost considerations require a modest number of runs.
3. Modeling flexibility is desirable

   U-Bridge designs allow for fitting both polynomial and Gaussian Process (GP) models.

When not to use U-Bridge Designs

1. The response data are deterministic.
2. Replication is necessary to control the variance

   That is, when the noise is very large compared to the signal

Recommendations for using U-Bridge Designs

1. U-Bridge designs have a lot of significant digits. Feel free to round to a few digits if precise control of factors is difficult
2. If f is the number of factors, the number of parameters in a full quadratic model is $p = ((f+1)(f+2))/2$

   Example: If $f=4$ $p = 5*6/2 = 15$. We recommend that the number of runs, n, should exceed $p + 5 = 15+5 = 20$ or more
3. Augmenting approaches

   If the polynomial fits well augment by choosing runs where the prediction variance is high.

   If bias is the main concern, augment with points where the difference between the GP and polynomial models is large in magnitude

Summary

1. U-Bridge designs allow for efficient polynomial estimation if there is noise in the response.
2. U-Bridge designs also allow for fitting GP models by avoiding replication in one-dimensional projections.
3. U-Bridge designs require relatively few runs, so they can be used early in a sequential experimentation program